

## Spatial Regression with Conditional Autoregressive (CAR) Errors for Annual Mean Relative Humidity in Peninsular Malaysia

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### ABSTRACT

Modelling observed meteorological elements can be useful. For instance, modelling rainfall has been an interest for many researchers. In a previous research, trend surface analysis was used and it was indicated that the residuals might spatially be correlated. When dealing with spatial data, any modelling technique should take spatial correlation into consideration. Hence, in this project, fitting of spatial regression models, with spatially correlated errors to the annual mean relative humidity observed in Peninsular Malaysia, is illustrated. The data used in this study comprised of the annual mean relative humidity for the year 2000-2004, observed at twenty principal meteorological stations distributed throughout Peninsular Malaysia. The modelling process was done using the S-plus Spatial Statistics Module. A total of twelve models were considered in this study and the selection of the model was based on the  $p$ -value. It was found that a possible appropriate model for the annual mean relative humidity should include an intercept and a term of the longitude as covariate, together with a conditional autoregressive error structure. The significance of the coefficient of the covariate and spatial parameter was established using the Likelihood Ratio Test. The usefulness of the proposed model is that it could be used to estimate the annual mean relative humidity at places where observations were not recorded and also for prediction. Some other potential models incorporating the latitude covariate have also been proposed as viable alternatives.

**Keywords:** Relative humidity, environment, spatial regression, Simultaneous Autoregressive errors, Conditional Autoregressive errors

### INTRODUCTION

Climatic changes have been occurring for the past hundred years or so, and some of these changes are attributable to human activities like deforestation, changes in land use, etc. Many meteorological elements are observed at meteorological stations like rainfall, sunshine, air temperature, radiation, atmospheric pressure, wind velocity, evaporation and the like. One of the elements observed or recorded at a meteorological station is relative humidity which is defined as the ratio of the mass of water vapour actually present in unit volume of the air to that required to saturate it at the same temperature. Relative humidity is usually expressed in percentage.

Modelling the observed meteorological elements can be useful. For instance, modelling rainfall has been an interest for many researchers. Le Cam (1961), Waymire and Gupta (1981) and Cox and Isham (1988) have adopted point process based models. Time series models, bivariate models and variogram analysis have been presented by Smith (1994). One of the early studies on rainfall

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Received: 5 July 2007

Accepted: 13 April 2009

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analysis in Malaysia (Todorov and Abraham, 1982) used the traditional statistical methods to identify dry areas and variability in the annual rainfall. A modelling technique, known as the Trend Surface Analysis, was used to fit models for the agricultural land value data for 1977-8 in Iowa (Cliff and Ord, 1981) and to study the forest landscape patterns (Jin-Ping, Guo and Yang, Xiao, 1999). More recently, the trend surface analysis of the annual rainfall in Peninsular Malaysia was undertaken by Isthriyagy (2001). It was suggested that if the residuals were spatially autocorrelated, the modelling procedure should then take into consideration the correlation reflecting the spatial structure. In fact, whenever dealing with spatial data, it is vital to be thoughtful of the spatial correlation.

Shitan (2004) modelled the annual mean relative humidity with Simultaneous Autoregressive (SAR) errors for the year 2001. Another error structure is the Conditional Autoregressive (CAR) type and Shitan *et. al.* (2005) fitted a spatial regression with the CAR errors to the same data set. In both these studies, it was concluded that an appropriate model for the annual mean relative humidity should include an intercept and a term of the longitude (i.e.  $Y_i = \beta_{00} + \beta_{01}x_2 + \varepsilon_i$ ). There is also another common error structure known as the Moving Average (MA) errors. Shitan and Kok (2007) have also modelled the same data set with the MA error structure. In their study, it was concluded in that the quadratic model  $Y_i = \beta_{00} + \beta_{01}x_2 + \beta_{20}x_1^2 + \varepsilon_i$  as an appropriate one since the linear model was not computable due to the numerical difficulties for the MA error structure. However, in all these studies, the data set used was only for one year. The research could be strengthened by considering more data and for this reason, the data for five years (2000 to 2004) were used in the present research.

In this project, the modelling of the annual mean relative humidity was concentrated on and the objective was to fit and illustrate the spatial regression with the Conditional Autoregressive (CAR) errors covariance structure. In Section 2, the methodology of the study is described, followed by the results in Section 3 and finally the conclusions are drawn in Section 4.

## METHODOLOGY

The data set for this study included the annual mean relative humidity (Table 1) from 2000 to 2004, observed at twenty (20) principal meteorological stations distributed throughout Peninsular Malaysia and operated by the Malaysian Meteorological Service. These principal meteorological stations are located at Batu Pahat, Kluang, Mersing, Senai, Alor Setar, Langkawi, Kota Bharu, Kuala Krai, Melaka, Kuantan, Temerloh, Ipoh, Lubok Merbau, Sitiawan, Chuping, Bayan Lepas, Butterworth, Subang, Petaling Jaya and Kuala Terengganu. Cameron Highlands was excluded from this study because it is situated at 1,545 metres above the mean sea level, which is located much higher than the other stations, where height ranges only from 3 to 88 metres.

The psychrometer or hygrometer, which is a combination of dry bulb and wet bulb thermometers, was used to obtain the relative humidity necessitated for the study. If the air is dry, evaporation occurs rapidly, and this thus lowers the reading of the wet bulb thermometer; the differences in the reading for the dry bulb and wet bulb were used to compute the relative humidity. For the continuously recording relative humidity, on the other hand, the standard instrument is the hair hygrograph where the humidity sensitive element is a bundle of hair which has the property of altering in length with changes in the relative humidity. Both these instruments were kept in a Stevenson screen at the principal meteorological stations.

A class of models incorporating the correlation reflecting the spatial structure is of the form,  $Y_i = \mu_i + \varepsilon_i$ , where  $Y_i$  is the random variable at site  $i$ ,  $\mu_i$  is the mean at site  $i$  which is modelled in

TABLE 1  
Co-ordinates, annual mean relative humidity and neighbours  
of principal meteorological stations

	Meteorological Station	Latitude (degrees North)	Longitude (degrees East)	Annual Mean Relative Humidity (%)					Neighbours
				2000	2001	2002	2003	2004	
1	Batu Pahat	1.917	103.00	87.9	87.3	86.8	87.8	87.1	Kluang, Mersing, Senai, Melaka
2	Kluang	2.067	103.42	86.5	86.3	85.1	86.4	85.2	Batu Pahat, Mersing, Senai
3	Mersing	2.333	103.83	87.9	88.3	86.3	86.9	86.0	Kluang, Batu Pahat, Senai
4	Senai	1.600	103.63	86.1	86.2	84.7	85.3	84.6	Kluang, Batu Pahat, Mersing
5	Alor Setar	6.117	100.42	83.7	83.0	78.7	80.1	79.0	Chuping, Langkawi, Butterworth, Bayan Lepas
6	Langkawi	6.333	99.83	79.5	79.6	77.0	78.1	78.1	Chuping, Alor Setar
7	Kota Bharu	6.100	102.25	81.9	82.9	81.5	81.8	81.0	Kuala Krai
8	Kuala Krai	5.533	102.22	86.1	85.2	84.4	85.7	84.9	Kota Bharu, Kuala Terengganu
9	Melaka	2.333	102.28	82.4	82.4	80.1	81.4	80.5	Petaling Jaya, Batu Pahat
10	Kuantan	3.800	103.33	85.2	84.4	81.4	83.7	85.1	Temerloh
11	Temerloh	3.450	102.53	85.1	84.2	82.8	84.3	83.7	Petaling Jaya, Subang, Kuantan
12	Ipoh	4.567	101.05	83.3	83.7	81.6	81.9	80.9	Lubok Merbau, Sitiawan
13	Lubok Merbau	4.817	100.87	84.3	83.3	81.1	82.9	82.1	Ipoh, Butterworth, Bayan Lepas, Sitiawan
14	Sitiawan	4.217	100.72	84.3	84.0	83.5	85.3	81.7	Ipoh, Lubok Merbau
15	Chuping	6.500	100.25	83.6	82.9	81.7	82.6	81.7	Alor Setar, Langkawi
16	Bayan Lepas	5.200	100.18	80.9	80.5	75.1	77.1	77.3	Butterworth, Alor Setar, Lubok Merbau
17	Butterworth	5.400	100.35	82.3	80.9	79.5	81.3	80.8	Bayan Lepas, Alor Setar, Lubok Merbau
18	Subang	3.200	101.58	79.2	79.2	77.9	80.0	79.8	Petaling Jaya, Temerloh
19	Petaling Jaya	3.083	101.67	77.8	79.0	77.0	78.2	77.4	Subang, Melaka, Temerloh
20	Kuala Terengganu	5.333	103.12	84.3	84.0	81.3	82.4	81.1	Kuala Krai

terms of the covariates and  $\varepsilon_i$  the random error terms. Furthermore,  $\varepsilon_{ii}$  could be allowed to be a function of the neighbouring sites as:

$$\varepsilon_i = \sum_{\substack{j=1 \\ j \neq i}}^n g_{ij} \varepsilon_j + \delta_i \quad i = 1, 2, \dots, n \tag{1}$$

with  $\{g_{ij}\}$  a sequence of constants,  $\{\delta_i\}$  a sequence uncorrelated errors with  $E(\delta_i) = 0$  and  $\text{Var}(\delta_i) = \sigma^2$ . This is what is known as a Simultaneous Autoregressive (SAR) Model (see Cliff and Ord, 1981).

This model can be written in the matrix forms as,  $\varepsilon = \mathbf{G} \varepsilon + \boldsymbol{\delta}$ , where vector  $\varepsilon^T = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$ , vector  $\boldsymbol{\delta}^T = (\delta_1, \delta_2, \dots, \delta_n)$ ,  $\varepsilon \sim \text{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$ ,  $\boldsymbol{\delta} \sim \text{MVN}(\mathbf{0}, \sigma^2 \mathbf{I})$  and the matrix  $\mathbf{G}$  is given as follows:

$$\mathbf{G} = \begin{bmatrix} 0 & g_{12} & g_{13} & \dots & g_{1n} \\ g_{21} & 0 & g_{23} & \dots & g_{2n} \\ g_{31} & g_{32} & 0 & \dots & g_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \dots & \dots & 0 \end{bmatrix} \tag{2}$$

If we let  $\varepsilon_i^* = \{\varepsilon_j, j \neq i\}$ , where  $\varepsilon_i^*$  denotes  $\varepsilon$  after deletion of  $\varepsilon_i$ , the Conditional Autoregressive (CAR) Model is then:

$$E(\varepsilon_i | \varepsilon_i^*) = \sum_{\substack{j=1 \\ j \neq i}}^n g_{ij} \varepsilon_j \text{ and } \text{Var}(\varepsilon_i | \varepsilon_i^*) = \sigma_i^2 \text{ for } i = 1, 2, \dots, n \tag{3}$$

When each conditional distribution is normal, the matrix form of the joint distribution becomes  $\varepsilon \sim \text{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$ , where  $\boldsymbol{\Sigma}^{-1} = \mathbf{D}(\mathbf{I}-\mathbf{G})$ ,  $\mathbf{D}^{-1} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2)$ ,  $\mathbf{G}$  is given in equation (2), where  $\mathbf{D}\mathbf{G}$  and  $\mathbf{G}$  must necessarily be symmetric for the CAR model.

Since  $g_{ij}$  are constants which need estimation and too many of these constants are to be estimated, some simplifications can be made by allowing  $\mathbf{G} = \rho \mathbf{W}$ , where  $\rho$  is an unknown constant which can be estimated for a given data set, and

$$\mathbf{W} = \begin{bmatrix} 0 & w_{12} & w_{13} & \dots & w_{1n} \\ w_{21} & 0 & w_{23} & \dots & w_{2n} \\ w_{31} & w_{32} & 0 & \dots & w_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & \dots & 0 \end{bmatrix}, \tag{4}$$

is a matrix of known weights. Suppose we let  $\sigma_i^2 = \sigma^2$  for all  $i$ , the covariance matrix,  $\boldsymbol{\Sigma}$  would then be given as  $\sigma^2(\mathbf{I}-\rho \mathbf{W})^{-1}$  for the CAR model.

To obtain the weights, the researcher first had to ascertain or define the neighbouring sites and then worked out the weights. For this study, the neighbours for a given meteorological station were defined as all meteorological stations located within a radius of one(1) degree from the station of interest. The weights,  $w_{ij} = 1$ , if stations  $i$  and  $j$  were neighbours and  $w_{ij} = 0$ , if otherwise. The neighbours of the twenty meteorological stations considered in this study are listed in Table 1.

Various models of increasing complexity (as discussed in the results section in Section 3), were fitted to the relative humidity data and the modelling process was done using the S-plus Spatial Statistics Module (Kaluzny *et al.*, 1998).

To evaluate between the competing models, the test statistic (Cressie, 1993) used in this study was:

$$U^2 = 2\left(\frac{n-p-r}{n}\right)(L_p - L_{p+r}) \sim \chi^2(r), \tag{5}$$

where  $n$  is the number of data points,  $p$  is the number of parameters estimated,  $r$  is the additional number of the parameters estimated,  $L_p$  is the negative log likelihood for the smaller model and  $L_{p+r}$  is the negative log likelihood for the larger model.

The log likelihood function for the CAR model is given by:

$$-\frac{n}{2} \log(2\pi) - \frac{n}{2} \log(\sigma^2) + \frac{1}{2} \log |\mathbf{I} - \rho \mathbf{W}| - \frac{1}{2\sigma^2} \boldsymbol{\varepsilon}^T (\mathbf{I} - \rho \mathbf{W}) \boldsymbol{\varepsilon} \tag{6}$$

To determine whether any of the coefficients of the covariates were significant or not, the Likelihood Ratio Test given as  $-2\log\lambda \sim \chi^2(k)$  was used (*see* Maddala, 1989, 84), where  $k$  is the number of restrictions and:

$$\lambda = \frac{\text{maximum of Likelihood under restriction}}{\text{maximum of Likelihood without restriction}} \tag{7}$$

### RESULTS

A scatter plot of the annual mean relative humidity versus the latitude and longitude are shown respectively in *Figs. 1* and *2*. From these figures, it is clearly shown that there is some sort of relationship between the mean relative humidity and geographical co-ordinates.

The test for the spatial correlation was also conducted using the Moran and Geary Statistics (Cliff and Ord, 1981). The Moran spatial correlation was found to be 0.6231, with the standard error of 0.08828. The computed  $z$ -statistic value was 7.173 and a  $p$ -value of  $7.329 \times 10^{-13}$ . The Geary spatial correlation value was 0.5128, with the standard error of 0.09752. The computed  $z$ -statistic value was  $-4.996$  and a  $p$ -value of  $5.844 \times 10^{-7}$ . These tests indicated that the observations were spatially correlated due to the extremely small  $p$ -value, thereby rejecting the null hypothesis of the no spatial correlation. Hence, various models of increasing complexity were fitted into the data, as follows:

Let  $Y_i$  be the annual mean relative humidity recorded at station  $i$ ,  $x_1$  be the latitude and  $x_2$  be the longitude position of the stations.

The models considered in this study were:

$$Y_i = \beta_{00} + \varepsilon_i, \tag{Model 1}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \varepsilon_i, \tag{Model 2}$$

$$Y_i = \beta_{00} + \beta_{01}x_2 + \varepsilon_i, \tag{Model 3}$$

$$Y_i = \beta_{00} + \beta_{20}x_1^2 + \varepsilon_i, \tag{Model 4}$$

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \varepsilon_i, \tag{Model 5}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \varepsilon_i, \tag{Model 6}$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{20}x_1^2 + \varepsilon_i, \tag{Model 7}$$

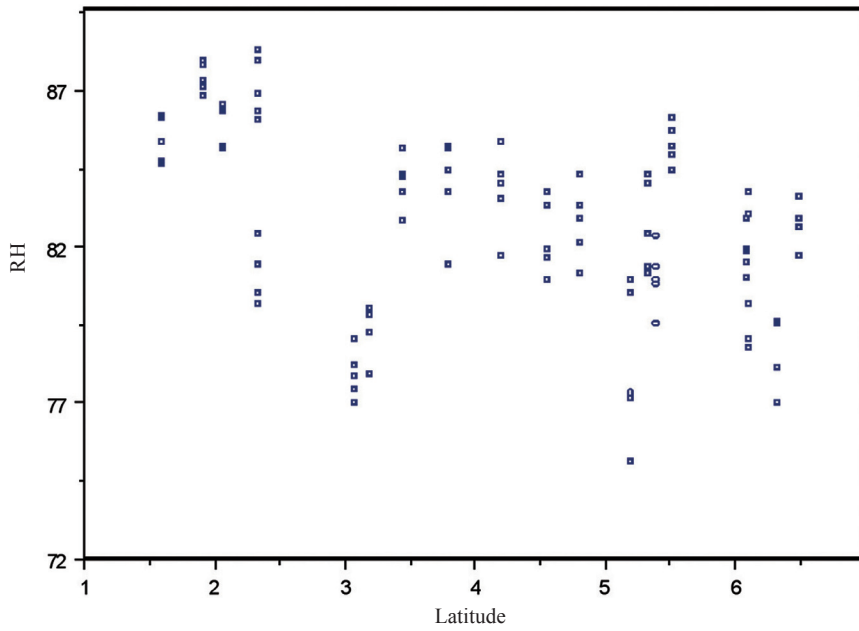


Fig. 1: Plot of annual mean relative humidity vs. latitude

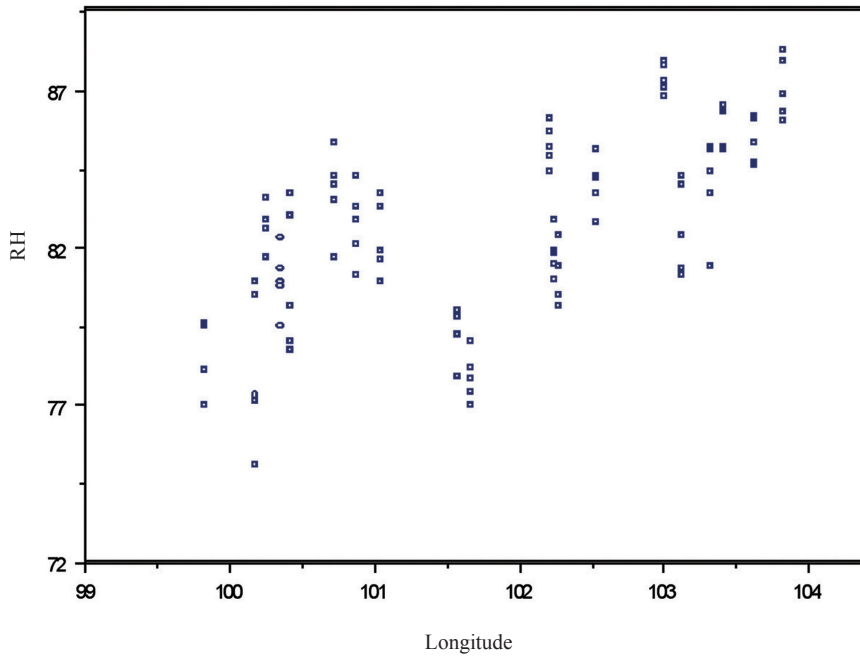


Fig. 2: Plot of annual mean relative humidity vs. longitude

$$Y_i = \beta_{00} + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \varepsilon_i, \quad (\text{Model 8})$$

$$Y_i = \beta_{00} + \beta_{01}x_2 + \beta_{20}x_1^2 + \varepsilon_i, \quad (\text{Model 9})$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \beta_{20}x_1^2 + \varepsilon_i, \quad (\text{Model 10})$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \varepsilon_i, \quad (\text{Model 11})$$

$$Y_i = \beta_{00} + \beta_{10}x_1 + \beta_{01}x_2 + \beta_{11}x_1x_2 + \beta_{20}x_1^2 + \varepsilon_i, \quad (\text{Model 12})$$

The parameter estimates of the fitted models for the *CAR* error structures in the present study are given in Table 2. Using equation (5), the test statistic  $U^2$  were computed for the various models considered in this study and are also tabulated in Table 2, together with the  $p$ -values.

Based on the data presented in Table 2, it could be observed that the estimated parameter coefficients takes on a wide variety of values, which were both positive and negative. The estimate for  $\sigma^2$  remained in the interval of 4.233 to 5.045, while for  $\rho$  it is in the range between 0.182 and 0.312 for the models considered in this study. The log likelihood remained in the vicinity of  $-318.74$  to  $-302.46$ , while  $U^2$  did not exceed 30.281.

However, the most crucial thing which needs to be observed in Table 2 is the  $p$ -values, which range from  $7.58 \times 10^{-7}$  to 0.759. The  $p$  value indicates whether or not a particular model significantly differs from the null model (Model 1). Then, a smaller  $p$ -value would clearly assist in the selection of a model. Among the models considered in this study, model  $Y_i = \beta_{00} + \beta_{01}x_2 + \varepsilon_i$  (Model 3) has the smallest  $p$ -value of  $7.58 \times 10^{-7}$  and it is therefore highly significant at 0.001 level. The significance of the coefficient of the longitude covariate was established by the Likelihood Ratio Test, which gave the value,  $\chi^2 = 25.487$  with 1 degree of freedom and  $p$ -value of  $4.453 \times 10^{-7}$ . Hence, this coefficient is highly significant at 0.001 level, indicating that  $\beta_{01}$  is not zero. In order to test the significance of  $\rho$ , the Likelihood Ratio Test gave a value,  $\chi^2 = 11.049$ , with 1 degree of freedom and  $p$  value of 0.001. This is also highly significant at the 0.01 level.

Following closely behind Model 3 are Models 9, 12, 6, 10 and 11 which have the  $p$ -values of  $4.20 \times 10^{-6}$ ,  $4.29 \times 10^{-6}$ ,  $4.58 \times 10^{-6}$ ,  $1.64 \times 10^{-5}$  and  $5.67 \times 10^{-4}$ , respectively. The other models can be safely discarded.

## CONCLUSIONS

The objective of this research was to fit and illustrate a spatial regression modelling which takes into account the spatial correlation amongst its neighbours. It was found that model  $Y_i = \beta_{00} + \beta_{01}x_2 + \varepsilon_i$  (Model 3) is an appropriate one to be used as it has the smallest  $p$  value when compared to the null model (Model 1). Using more data, this consolidates the previous studies undertaken. The coefficient  $\beta_{01}$  was also found to be significant. The parameter  $\rho$  was highly significant at 0.01 level, and this explains the importance of taking the spatial correlation between the neighbouring sites into consideration in the modelling process. The usefulness of this model is that it is simple and it will help to estimate the mean annual relative humidity at places where no observations were recorded for the year 2000 to 2004. It would therefore be useful for predicting future values.

There are also other potential models, including Models 9, 12, 6, 10 and 11 which also have small  $p$  values. In any future study, these potential models need to be given due consideration as they are viable alternatives.

Similar further studies can be undertaken for the data observed for other years apart from the ones considered in this research. Different neighbourhood structures and weights can also be attempted in any future study. Alternatively, further research can be done to fit the spatial regression models with Moving Average (*MA*) errors and make comparison with the proposed model in this research.

TABLE 2  
Results of fitted models for the CAR error structures

	Estimated parameter Coefficients	$\hat{\sigma}$	$\hat{\rho}$	Log Likelihood	$U^2$	$p$ -value
Model 1	$\hat{\beta}_{00} = 82.205$	4.837	0.289	- 318.74	-	-
Model 2	$\hat{\beta}_{00} = 83.522$ $\hat{\beta}_{10} = -0.275$	5.045	0.275	- 318.48	0.499	0.480
Model 3	$\hat{\beta}_{00} = -64.124$ $\hat{\beta}_{01} = 1.438$	4.241	0.225	- 306.00	24.461	$7.58 \times 10^{-7}$
Model 4	$\hat{\beta}_{00} = 82.912$ $\hat{\beta}_{20} = -0.029$	5.000	0.279	- 318.46	0.538	0.463
Model 5	$\hat{\beta}_{00} = 83.154$ $\hat{\beta}_{11} = -0.002$	5.005	0.280	- 318.62	0.230	0.632
Model 6	$\hat{\beta}_{00} = -73.545$ $\hat{\beta}_{10} = 0.153$ $\hat{\beta}_{01} = 1.524$	4.234	0.232	- 305.80	24.586	$4.58 \times 10^{-6}$
Model 7	$\hat{\beta}_{00} = 81.972$ $\hat{\beta}_{10} = 0.397$ $\hat{\beta}_{20} = -0.068$	4.977	0.285	- 318.45	0.551	0.759
Model 8	$\hat{\beta}_{00} = 73.167$ $\hat{\beta}_{11} = 0.038$ $\hat{\beta}_{20} = -0.396$	4.310	0.312	- 317.22	2.888	0.236
Model 9	$\hat{\beta}_{00} = -75.000$ $\hat{\beta}_{01} = 1.540$ $\hat{\beta}_{20} = 0.021$	4.234	0.230	- 305.71	24.757	$4.20 \times 10^{-6}$
Model 10	$\hat{\beta}_{00} = -71.768$ $\hat{\beta}_{10} = -0.803$ $\hat{\beta}_{01} = 1.524$ $\hat{\beta}_{20} = 0.111$	4.327	0.216	- 305.51	24.872	$1.64 \times 10^{-5}$
Model 11	$\hat{\beta}_{00} = 86.620$ $\hat{\beta}_{10} = -29.780$ $\hat{\beta}_{11} = 0.271$ $\hat{\beta}_{20} = 0.249$	4.586	0.235	- 309.45	17.465	$5.67 \times 10^{-4}$
Model 12	$\hat{\beta}_{00} = -335.944$ $\hat{\beta}_{10} = 56.962$ $\hat{\beta}_{01} = 4.059$ $\hat{\beta}_{11} = -0.540$ $\hat{\beta}_{20} = -0.198$	4.233	0.182	- 302.46	30.281	$4.29 \times 10^{-6}$



### ACKNOWLEDGEMENTS

The researchers would like to thank the reviewers and the editor for their useful comments and valuable suggestions to improve the quality of the paper. We would also express our thanks to the Department of Mathematics and the Institute of Mathematical Research, Universiti Putra Malaysia for their support.

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